

In their report, Cerioli et al. (13) are able to circumvent these issues by preassessing the conditions under which the NBL should emerge in the context of international trade—that is, the conditions under which the subsequent inferences can be trusted. These authors actually pioneer the application of the NBL to the context of international trade. Why is international trade an important area of application? Because of the money. Big money. To give a sense of the scale, the EU accounts for around 15% of the world's trade in goods, translating to over €1,800 billion on imports and a similar quantity of exports in 2017 alone, with a positive balance between the two of over €20 billion (17). Being able to detect fraudulent behavior—including underreported goods—is therefore of paramount importance (18). Devising a robust and accurate statistical method that could be easily embedded online to automatically monitor transaction activities and flag suspicious ones would tick all of the boxes (i.e., reliability, flexibility, efficiency, and cost-effectiveness). This approach also aligns with the overall modernization strategy currently pursued by the World Customs Organization (19).

Cerioli et al. (13) successfully address the two main challenges that might otherwise preclude a realistic detection of fraud via NBL analysis of individual traders. First, the authors establish solid conditions for the validity of the NBL in the field of international trade data—an essential step for the implementation of large-scale, automatic monitoring processes. Second, they find approximations and corrections to the adequate test statistics needed to scrutinize fraud in those instances in which the NBL is not actually expected to hold. Importantly, the authors are therefore able to discriminate two types of nonconformance to the NBL: cases related to data fabrication and fraud versus the so-called false-positives, which are legitimate deviations that emerge, for example, for traders who operate on a limited number of products (so that there is not enough variability in their transactions for the NBL to emerge in the first place). As a result, traders can be classified into three groups: (i) legitimate traders whose activity conforms to the NBL, (ii) traders whose activity is still legitimate but does not conform to the NBL due to controlled factors, and (iii) traders whose activity does not conform to the NBL even when it should, probably due to data fabrication (see Fig. 1 for an illustration).

On a technical level, Cerioli et al. (13) initially argue that the sequence of transaction values for a particular trader complies with Hill's (3) central limit theorem hypothesis. By generating synthetic transactions of "idealized" traders, the authors can statistically assess the conditions where the NBL holds and in what circumstances classic χ^2 tests can therefore be applied. An important result is that the NBL breaks down as the theoretical expected distribution when the number of different traded goods, m , is much smaller than the total number of transactions, n ($m \ll n$), so that in those instances, a blind χ^2 test is not recommended.

One of the important aspects of Cerioli et al.'s report (13) is that they model transaction data via a trader-specific contamination model composed by linearly interpolating a mixture of two

distributions for the leading digit: a legitimate one (usually the NBL), which is parametrically dependent on n and m ; and a contaminant distribution, which models the effect of data manipulation. The null hypothesis (legitimate trader) is equivalent to having a null coefficient on the contaminant distribution; however, this scenario is equivalent to conformance to the NBL only when the transaction data fulfill the criteria discussed above. In those cases in which the expected distribution for nonfraudsters deviates from the NBL for legitimate reasons, Cerioli et al. can plug in the actual expected distribution as the legitimate one and can deduce, among other things, the correct distribution of the test statistic that shall be used to reject the null hypothesis of a legitimate transaction pattern.

Cerioli et al.'s paper provides a principled framework for goodness-of-fit testing of the NBL for antifraud purposes, with a focus on customs data.

Validation and calibration of the theoretical framework was possible thanks to access to real data of import/exports declared by national traders and shipping agents using the Single Administrative Document form (data were provided by the Italian Customs Agency and by the customs office of another European Union member state not disclosed for its specific confidentiality policy). Results overall show that Cerioli et al.'s (13) methodology can flag fraudsters as well as deflag traders whose activity deviates from expected the NBL due to legitimate reasons. The authors discuss a particularly illuminating case of a trader extracted from an archive of fraudulent declarations provided by the Italian Customs Agency, whose fraudulent behavior was discovered only after substantial investigation of two of the declarations. In this case, a standard protocol based on robust regression techniques aiming at the automatic detection of value frauds in customs data did not provide clear evidence of substantial undervaluation or of other major anomalies. Cerioli et al.'s analysis, on the other hand, produced a strong signal of contamination of the digit distribution for this trader, and their statistical analysis safely concluded the presence of fraudulent manipulation.

In conclusion, Cerioli et al.'s paper (13) provides a principled framework for goodness-of-fit testing of the NBL for antifraud purposes, with a focus on customs data. This methodology has the potential to be embedded—probably in combination with more standard and model-free approaches (6, 12)—in real international trade antifraud protocols and audits in the near future. In this respect, a web application (13) developed with the purpose of assisting customs officers and auditors in the screening task has already been set in place.

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